Counting Principle - 3

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Circular Permutation :→

The arrangements of n persons around a table can be done in $\lfloor n-1$ ways.

The arrangements can be done in two directions

(i) Clockwise (ii) Anticlockwise.

When directions are not considerend then the no. of arrangements can be done in $\frac{1}{2}\lfloor n-1$

e.g necklace

Q:→ In how many ways 6 beads of different colours form a necklace.

sol:→ Req. no. of necklaces $= \frac{1}{2}\lfloor 6-1 = \frac{\lfloor 5}{2} = \frac{120}{2} = 60$

Q:→ Four persons A, B, C, D are to be seated at a circular table. In how many ways can they be seated?

sol:→ No. of ways $= \lfloor n-1 = \lfloor 4-1 = \lfloor 3 = 6$

Q:→ In how many ways can 5 boys and 5 girls be seated at a round table, so that no two girls sit together?

sol:→ Let the boys be seated leaving one seat vacant in between each of two boys. ($\because$ no two girls to sit together)

This can be done in $\lfloor 5-1 = \lfloor 4$ ways

Now 5 girls can be arranged in 5 vacant seats in $^5P_5 = \lfloor 5$

$\therefore$ Reqd no. ways $= \lfloor 4 \times \lfloor 5$

$= 24 \times 120$

$\therefore$ Reqd no. ways $= \lfloor 4 \times \lfloor 5$
$= 24 \times 120$
$= 2880$

Combination

An unordered selection of r objects from n distinct objects.

$${}^{n}C_r = \binom{n}{r} = C(n, r) = \frac{\lfloor n}{\lfloor r \lfloor n-r} \quad ; \; 0 \leq r \leq n$$

Properties

(i) ${}^{n}C_r = {}^{n}C_{n-r}$

(ii) ${}^{n}C_n = {}^{n}C_0 = 1$

(iii) ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

(iv) ${}^{n}C_0 + {}^{n}C_1 + \dots + {}^{n}C_n = 2^n$

(vi) ${}^{n}C_r = \dfrac{{}^{n}P_r}{\lfloor r}$

Ex:→

(i) Verify that ${}^{8}C_4 + {}^{8}C_3 = {}^{9}C_4$

(ii) Find n if ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$ — Ans $n = 6$

(iii) If ${}^{n}C_9 = {}^{n}C_8$ find ${}^{n}C_{17}$ — Ans 1

(iv) If ${}^{n-1}C_r : {}^{n}C_r : {}^{n+1}C_r = 6 : 9 : 13$ find n and r

Ans $n = 12$, $r = 4$

(v) If ${}^{n}C_x = 56$ and ${}^{n}P_x = 336$ find n and x

Ans $n = 8$, $x = 3$

Sol

$${}^{n}C_x = 56 \Rightarrow \frac{\lfloor n}{\lfloor x \lfloor n-x} = 56 \quad \text{—— (1)}$$

$${}^{n}P_x = 336 \Rightarrow \frac{\lfloor n}{\lfloor n-x} = 336 \quad \text{—— (2)}$$

$$^{n}P_x = 336 \Rightarrow \frac{n!}{(n-x)!} = 336 \quad \text{—(2)}$$

(2) ÷ (1)

$$\frac{\dfrac{\cancel{n!}}{\cancel{(n-x)!}}}{\dfrac{\cancel{n!}}{x!\,\cancel{(n-x)!}}} = \frac{336}{56}$$

$$x! = 6 = 3! \Rightarrow x = 3$$

As $\frac{n!}{(n-x)!} = 336$

$$\Rightarrow \frac{n!}{(n-3)!} = 336$$

$$\Rightarrow n(n-1)(n-2) = 336$$
$$= 8 \times 7 \times 6$$

$$\Rightarrow n = 6$$

2	336
2	168
2	84
2	42
3	21
	7

Practical Problems on Combinations :→

Q1 :→ In how many ways can a student choose a programme of 5 courses out of 9 courses and 2 courses are compulsary for every student ?

sol:→ Total courses available = 9

No. of courses to be taken = 5

∵ 2 courses are compulsary

∴ student has to select 3 courses out of 7 courses.

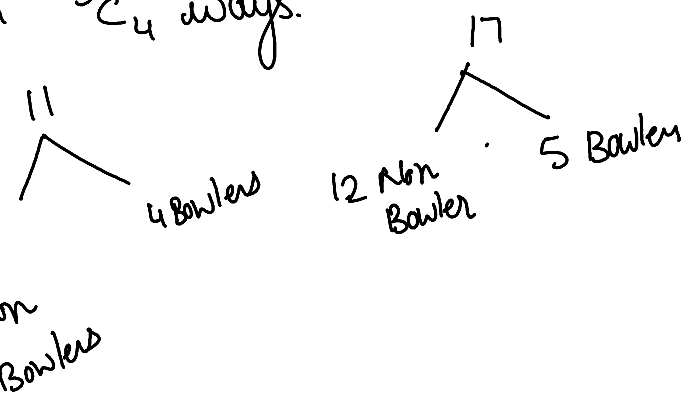
This can be done by $^{7}C_3 = \frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5}{6} = 35$

Q2:→ In how many ways can we select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of eleven must include 4 bowlers?

sol:→ No. of bowlers = 5

" " Non bowlers = 12

7 nonbowlers out of 12 nonbowlers players can be selected in $^{12}C_7$ ways and 4 bowlers out of 5 bowlers can be selected in $^5C_4$ ways.



∴ Reqd no. of teams $= {}^{12}C_7 \times {}^5C_4$

$$= \frac{\lfloor 12}{\lfloor 7 \cancel{\lfloor 5}} \times \frac{\cancel{\lfloor 5}}{\lfloor 4 \lfloor 1}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}$$

$$= 3960$$

Q:→ A group consist of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

i) no girl ii) atleast one boy and one girl

iii) atleast three girls

sol

7 Boys	4 Girls
5	0

sol

5	0

(i) No girl

Req. no. of ways $= {}^7C_5 \times {}^4C_0 = \frac{\lfloor 7}{\lfloor 5 \lfloor 2} \times 1 = \frac{7\times 6}{2} = 21$

(ii) Atleast one boy and one girl

7 Boys	4 Girls
1	4
2	3
3	2
4	1

Req. No. of ways $= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140$

$= 441$

iii) Atleast three girls

7 Boys	4 Girls
2	3
1	4

Req. No. of ways

$= {}^7C_2 \times {}^4C_3 + {}^7C_1 \times {}^4C_4$

$= 84 + 7$

$= 91$